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QUESTIONS AND DISCUSSIONS.

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A number of questions have been standing in this department for some time with no replies or with replies which still leave something to be desired in the way of completeness or finality. Several of these are reprinted below, with the intention of directing our readers' interest to them again, and stimulating replies. To those which have already received some attention short notes are appended, indicating the extent to which they still remain open for consideration.

QUESTIONS.

15 [*1914*, 278; *1916*, 353; *1919*, 68; *1920*, 114, 361]. In the *Proceedings of the Royal Society of Edinburgh*, vol. 7, p. 144, in some mathematical notes by Professor P. G. Tait, it is stated:

"If $x^3 + y^3 = z^3$, then $(x^3 + z^3)^3 y^3 + (x^3 - y^3)^3 z^3 = (z^3 + y^3)^3 x^3$.

"This furnishes an easy proof of the impossibility of finding two integers the sum of whose cubes is a cube."

How does this "easy proof" follow?

A large number of replies to this question have been received; they are analyzed in the remarks by the editor, *1920*, 361. It seems necessary to emphasize again that what is desired is *not* a proof that if $x^3 + y^3 = z^3$, then $(x^3 + z^3)^3 y^3 + (x^3 - y^3)^3 z^3 = (z^3 + y^3)^3 x^3$—this is of course very easy; but a proof, by use of this theorem, that no two positive integers exist the sum of whose cubes is a cube.

21 [*1914*, 341; *1916*, 354; *1919*, 68, 239; *1920*, 114]. For the Diophantine equation

$$x^2 - y^3 = 17$$

there are known the following solutions:

$$\begin{array}{rrrrrrrrr} x = & 3, & 4, & 5, & 9, & 23, & 282, & 375, & 378661, \\ y = & -2, & -1, & 2, & 4, & 8, & 43, & 52, & 5234. \end{array}$$

One of our readers, who supplied the foregoing facts, desires to know the answers to the following questions: Are there other solutions of the given Diophantine equation? How may all the solutions of this equation be found by a systematic procedure?

In a reply to this question published in the MONTHLY for June, 1919, E. B. Escott showed how to obtain an infinite number of rational solutions of the equation, and in particular all the integral solutions contained in the above list.

A number of references bearing on the question are given by Dickson, *History of the Theory of Numbers*, vol. 2, chapter 20, pp. 533–539. Of especial pertinence are the following:

Mordell, *Proceedings of the London Mathematical Society*, series 2, vol. 13, pp. 60–80; vol. 18, pp. v–vi.

Thue, *Crelle's Journal*, vol. 135, pp. 303–304.

30 [*1916*, 88, 354; *1920*, 114, 362]. A certain Normal University wishes to offer thirty-five hours of college mathematics for the benefit of high-school teachers. What should these courses be in order that, primarily, they may be of the greatest value to high-school teachers of mathematics and, secondarily, that they may furnish stimulus for a more extended pursuit of the subject?

Only one reply to this question has been received, *1920,* 362. It seems that more might be said on this question, and further replies are desired. Some idea of existing conditions may be obtained from the replies to another question in the MONTHLY, *1916,* 395.

34 [*1917,* 134, 341; *1920,* 114, 301, 405, 460; *1921,* 19]. Given the mixed integral and functional equation

$$\int_{x=0}^{x=h} f(x)dx = \frac{h}{6}\left[f(0) + 4f\left(\frac{h}{2}\right) + f(h)\right],$$

to determine the function $f(x)$. This equation is of rather fundamental practical value as it has to do with the most general solid whose volume is given by the prismatoid formula.

There has been a considerable amount of interesting discussion on this question, but its possibilities are by no means exhausted. A formulation of the sense in which the problem is to be interpreted was given by the editor, *1920,* 302. For continuous functions, analytic at the origin, the only solutions are cubic polynomials. It was shown by Gillespie, *1920,* 405, that the condition of analyticity at the origin may be replaced by the existence of six continuous derivatives near the origin. On the other hand remarks by Bennett, *1920,* 460, Weisner, *1921,* 20, and the editor, *1920,* 462, show that there exist solutions other than cubic polynomials, possessing continuous first derivatives. It would be of interest to know how little need be assumed beyond the continuity of the first derivative to restrict solutions to cubic polynomials.

The related question obtained by taking both limits of integration variable and suitably modifying the formula has reached a satisfactory answer with the proof by Ballantine, *1921,* 19, that the solution must be a cubic polynomial provided it is merely continuous.

35 [*1918,* 266; *1920,* 114]. Is the following theorem new or has it previously been published?

THEOREM. *If two parallel planes, π and π', cut sections from a cylindrical surface S and two spherical surfaces S_1 and S_2, and if the sum of the sections of S_2 is equal in area to the sum of the sections of S plus the sections of S_1, then the part of S_2 included between π and π' is equal in volume to the sum of the parts of S and S_1 included between π and π'.*

In communicating this question, E. O. Brower commented on the simplicity with which the theorem could be proved, which led him to wonder whether so simple a proposition had remained unnoticed.

39 [*1920,* 256]. There are certain problems in geometry which are simple in statement but can be reduced only to very complicated problems in transcendental analysis. Following are several examples of the type of problem in question.

1. What is the smallest plane area within which a given figure can be turned through a complete revolution? It is not implied that the figure should revolve about a fixed point, but merely that in the course of its motion the figure should have every possible orientation in the plane. The problem may be modified by considering only convex areas.

An interesting special case is that in which the given figure is a segment of a straight line. In this case it has been conjectured by Professors Osgood and Kubota that the smallest area may be bounded by a three-cusped hypocycloid; if we consider only convex areas, perhaps the result will be an equilateral triangle. I have no indication of a proof.

2. For every closed convex curve of area P there is an n-sided circumscribed polygon of least area Q and an n-sided inscribed polygon of greatest area R. For a fixed value of the integer n and for all closed convex curves, what is the upper limit of Q/P and what is the lower limit of R/P? I have succeeded only in proving that for the case $n = 3$, the upper limit of Q/P is 2.

3. Let the area of a given simple closed curve A be a. Remove from A the greatest possible area a_1 similar to another given simple closed curve B. From the remaining figure remove the greatest possible area a_2 similar to B. Continue this process indefinitely. Is it or is it not true that

$$a_1 + a_2 + a_3 + \cdots = a?$$

I have proved the statement to be true in the special case that A is convex and B is a circle.

4. Let a given closed convex curve K have the property that a given triangle whose angles are incommensurable with π can be revolved completely within K (see Part 1 of this question), always remaining inscribed in K. What may the curve K be? Can any other curve except a circle satisfy the conditions?

These questions were formulated by Professor S. Kakeya, who has treated some problems of similar nature in various papers in the *Science Reports* of the Tôhoku Imperial University. They involve considerations of decided difficulty in the analytic formulation of geometric relations. The special case mentioned in the first question has received some attention from W. B. Ford, *Bulletin of the American Mathematical Society*, vol. 27, p. 55.

40 [*1920*, 365]. How great emphasis is laid in freshman mathematics upon the elementary algebra of complex numbers? A recent paper by an eminent electrical engineer seems to indicate the need of a knowledge of this subject on the part of draftsmen and mechanics of very limited educational opportunities. The syllabus of the College Entrance Examination Board mentions the topic under the caption "Advanced Angebra," but the question papers call for only the slightest study of numbers of this type. Is Euler's theorem ($e^{i\vartheta} = \cos\vartheta + i\sin\vartheta$) usually presented in college algebra, or is it left to the calculus? Does the topic deserve greater emphasis than it usually gets, for the sake of applications in the field of periodic currents?

The discussion, "Complex Numbers in Advanced Algebra," by Mr. H. E. Webb, *1920*, 411, may be viewed as a reply to this question. There seems to be room for further discussion.

41 [*1920*, 365]. A reader asks for an elementary proof of the following two propositions in number theory, either of which can readily be obtained from the other:

Every positive integer of the form $8n + 3$ *is the sum of three odd squares.*

Every positive integer is the sum of not more than three triangular numbers.

Bachmann[1] states that these theorems have been proved only by the use of the theory of ternary quadratic forms, and refers to the discovery of the theorems by Gauss, and a comparatively simple proof by Dirichlet,[2] by means of this theory. A number of references are given by Dickson.[3] Apparently the theorem was first enunciated by Fermat.

42 [*1921*, 65]. In connection with the questions of Kakeya, Professor W. B. Ford is led to the following inquiry: A line-segment AB is to be moved in its plane to a new position $A'B'$. How should this be done in order that the area generated may, to the greatest extent possible, be passed over three times?

The questions of Kakeya here referred to are given as Question 39 above.

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[1] *Niedere Zahlentheorie*, Leipsic and Berlin, 1910, Teil 2, p. 325.

[2] *Crelle's Journal*, vol. 40, p. 228; *Liouville's Journal*, series 2, vol. 4, p. 233.

[3] *History of the Theory of Numbers*, vol. 2, Chapter 1.

Professor Ford has proved that if the generated area is to be passed over, to the greatest possible extent, but *two* times, AB should be rotated about the intersection of the perpendicular bisectors of AB and $A'B'$.

DISCUSSIONS.

Doctor Morris gives a device for a perpetual calendar, with an account of the theory on which it is constructed. A similar explanation will hold for any of the many forms of such calendars. It is believed that this article will be of interest, since explanations of the theory of perpetual calendars are seldom given.

Professor Bennett gives a set of simple identities from which a four-place logarithm table can readily be calculated. The method is probably new. It bears some relation to the plan used by Briggs (cf. *Encyclopædia Britannica,* 11th ed., vol. 16, pp. 875–876); but Briggs needed first to calculate and use explicitly the modulus, while Professor Bennett does not make any use of the modulus.

I. The Theory of Perpetual Calendars.

By Frank R. Morris, University of California.

A short discussion of adjustable calendars is given by Irwin Roman[1] in the Monthly, *1915,* 241. He describes one of the several hundred mechanical devices used in presenting the calendar of a given month of a given year, but he does not give the mathematical theory upon which practically all perpetual calendars are based. It is the purpose of this article to present this theory.

There are approximately 365.2422 days in a tropical year. If each calendar year contained 365 days, in the course of 400 years an error of 96.88 days would accumulate. Then if 97 days be added for each period of 400 years the error will be very small. In any 400 consecutive integers there are 100 numbers which are divisible by 4, and 3 which are divisible by 100 but not by 400. Hence the rule: *Each year which is divisible by* 4 *but not by* 100 *and each year which is divisible by* 400 *is a leap year with* 366 *days. All other years contain* 365 *days.* Since this rule was first introduced by Pope Gregory XIII in the year 1582, no calendar based upon it need antedate the sixteenth century. The calendar may be extended into the future as far as one may choose. However, it becomes inaccurate after a few thousand years, because it accumulates a day in about 3,000 years, and also because the tropical year varies slightly as the years go by. Other calendars, which are used in some parts of the world or have been used in the past, are built upon other rules but I shall discuss only that calendar

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[1] Mr. Roman's article begins with the statement, "So far as the writer has been able to learn, all perpetual or adjustable calendars are arranged so as to present the first day of the month as the first day of the week." He had evidently overlooked the most fruitful field for the study of perpetual calendars, viz., the *Official Gazette* of the United States Patent Office. During the past 25 years more than 100 calendars have been patented and more than a score of these present the days of the week in the natural order. For example see number 1048413, Dec. 24, 1912. [Doctor Morris's perpetual calendar was patented Nov. 26, 1918, number 1286058.—Editor.]